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Contractions of sigma models and integration of massive modes

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Abstract

We show how the integration of massive modes after a spontaneous symmetry breaking in a sigma model can often be interpreted as a contraction, induced by a group contraction, of the target space of the sigma model.

1 Introduction

In this talk we summarize the results of Ref. [1]. The main idea is to find a geometrical way of describing the integration of massive modes after a gauging of translational isometries. We consider examples of sigma models that are maximally symmetric spaces and so they have a solvable Lie group structure. We apply the theory of contractions of Lie algebras and groups to define deformations and contractions of the metric of the sigma model, and we find out for several examples that the geometrical interpretation of the integration is a generalized contraction.

In section 2 we make a review of the theory of contractions and explain how do we apply it to our examples. In section 3 we start with a simple example, and we see that an exact integration or truncation of a theory is related with a contracted Lie algebra that is isomorphic to the non contracted one. Then we describe a more complicated model where an Inönü-Wigner [2] contraction models the integration of massive modes after the gauge symmetry breaking . In section 4 we describe a model where instead a generalized or Weimar-Woods contraction is needed to model the integration. This last model correspond to a certain supregravity theory.

2 Contractions of Lie algebras, groups and symmetric spaces

Let $\mathfrak{g}$ be a finite dimensional Lie algebra with commutator $[\ ,\ ]$, and ϵ a real parameter.

Let $\phi_\epsilon : \mathfrak{g} \to \mathfrak{g}$ denote a family of linear maps parametrized by ϵ, such that they are non degenerate except possibly for $\epsilon = 0$.

The *deformed commutator* on the vector space $\mathfrak{g}$

$$[X,Y]_\epsilon = \phi_\epsilon^{-1}([\phi_\epsilon(X), \phi_\epsilon(Y)]), \qquad X, Y \in \mathfrak{g}$$

defines a Lie algebra $\mathfrak{g}_\epsilon \approx \mathfrak{g}$ except possibly for $\epsilon = 0$. Because of the non degeneracy of ϕ_ϵ at $\epsilon \neq 0$, the deformed Lie algebra defined by $[\ ,\ ]_\epsilon$ is isomorphic to the one defined by $[\ ,\ ]$.

If the limit $[X,Y]_c = \lim_{\epsilon\to 0}[X,Y]_\epsilon$ exists, then it defines a Lie algebra structure on the vector space $\mathfrak{g}$ denoted by $\mathfrak{g}_c$. It is a *contraction* of the Lie algebra $\mathfrak{g}$. In general, $\mathfrak{g}$ is not isomorphic to $\mathfrak{g}_c$.

These are *generalized Inönü-Wigner* or *Weimar-Woods* contractions. The conditions on ϕ_ϵ to have a well defined bracket $[\ ,\ ]_c$ are studied in detail in Ref. [2, 3].

The standard Inönü-Wigner contraction is obtained when $\mathfrak{g}$ is split as $\mathfrak{g} = \mathfrak{g}_1 + \mathfrak{g}_2$, with $\mathfrak{g}_1$ a subalgebra, and

$$\phi_\epsilon = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \epsilon\mathbb{1} \end{pmatrix}.$$

The result has the form $\mathfrak{g}_c = \mathfrak{g}_1 \ltimes \mathbb{R}^n$.

We can also contract a representation of a Lie algebra to a representation of the contracted Lie algebra: if W is a module for $\mathfrak{g}$ ($R : \mathfrak{g} \to \mathrm{End}W$), one has to find a linear map $\psi_\epsilon : W \to W$ such that the limit

$$R_c(X) = \lim_{\epsilon\to 0} \psi_\epsilon^{-1} \circ R(\phi_e(X)) \circ \psi_\epsilon$$

exists. Then we have a representation of $\mathfrak{g}_c$.

For example, for the standard Inönü-Wigner contraction, it is enough to split $W = W_1 + W_2$ where W_1 is a representation of $\mathfrak{g}_1$ and

$$\psi_\epsilon = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \epsilon\mathbb{1} \end{pmatrix}.$$

Then R_c is well defined and it is a representation of $\mathfrak{g}_c$.

For example, the adjoint representation always admits a contraction to the adjoint representation of the contracted algebra using $\psi_\epsilon = \phi_e$.

We will consider deformations and contractions of Lie groups and symmetric spaces via exponentiation of the corresponding Lie algebras. This will be enough for our purposes, since the models that we will study admit global exponential coordinates.

2.1 Symmetric spaces and metrics

Let G be a semisimple Lie group with Lie algebra $\mathfrak{g}$ and consider a Cartan decomposition $\mathfrak{g} = \mathfrak{h} + \mathfrak{p}$. Let H be the maximal compact subgroup of G; it has Lie algebra $\mathfrak{h}$.

We consider the principal bundle $G \to G/H$ and a local section (or *coset representative*) on it $L : U \subset_{\mathrm{open}} G/H \to G$.

The pull back of the *Maurer-Cartan form* on $U \subset_{\mathrm{open}} G/H$ decomposes as

$$L^{-1}dL = (L^{-1}dL)_{\mathfrak{h}} + (L^{-1}dL)_{\mathfrak{p}}.$$

Then the invariant metric on G/H is

$$\langle (L^{-1}dL)_{\mathfrak{p}}, (L^{-1}dL)_{\mathfrak{p}} \rangle.$$

$\langle\ ,\ \rangle$ is the *Cartan-Killing form* of $\mathfrak{g}$. (One can see for example Ref. [4] for the details).

By using a representation of $\mathfrak{g}_\epsilon$ and its exponential, we can compute the coset representative L_ϵ and an ϵ-dependent metric, that is, a deformation of the metric. The inner product $\langle\ ,\ \rangle$ is the Cartan-Killing form of $\mathfrak{g}$, so it is independent of ϵ. This implies that the metrics are not isometric. There is no change of coordinates taking one metric into the other, and the limit $\epsilon \to 0$ produces a non degenerate metric. We can have Einsten spaces that are deformed to non Einstein metrics, which shows that the deformation is non trivial, although the Lie algebras are isomorphic.

Using a Cartan Killing form depending on ϵ would not give a true deformation of the metric. Moreover, the limit $\epsilon \to 0$ will be degenerate, since the group becomes non semisimple.

The *Iwasawa* or KAN *decomposition* (here $K = H$, N is nilpotent and A abelian) assures us that the tangent space to the coset at the identity, $\mathfrak{p}$ has a solvable Lie algebra structure. In fact, $G/H \sim AN$ is itself a solvable Lie group inside G. We can consider contractions of this group instead of the whole group G, which gives us more possibilities.

3 First examples.

Exact truncation. We consider the symmetric space $\mathrm{SO}(1, 1+n)/\mathrm{SO}(1+n)$, whose solvable Lie algebra is given by the commutation relations

$$[H, Y_a] = Y_a, \quad a = 1, \dots n.$$

We choose the following coset representative $L = e^{u^a Y_a} e^{\varphi H}$, which is an element of the solvable group. The coordinates u^a, φ are *global coordinates* (for a proof see appendix in Ref. [1]). Using the technic explained in Section 2.1, one can compute the metric in these coordinates:

$$ds^2 = d\varphi^2 + e^{-2\varphi} \sum_{a=1}^{n} (du^a)^2.$$

It is immediate to see that the transformations $u^a \to u^a + c^a$, with $c^a \in \mathbb{R}$ are isometries of the metric, generated by Y^a. They are *translational isometries*.

Suppose that we have a sigma model with such metric. Let us *gauge* one of the translational isometries, say Y_n. This means that we introduce a gauge field $A = A_\mu dx^\mu$ and substitute du^n by the covariant differential $Du^n = du^n + gA$. We can equally use the gauge transformed connection $\hat{A} = A + \frac{1}{g}du^n$. Substituting this in the metric we obtain

$$ds^2 = d\varphi^2 + e^{-2\varphi}\sum_{a=1}^{n-1}(du^a)^2 + e^{-2\varphi}g^2\hat{A}^2.$$

We observe that the coordinates u^n have disappeared from the kinetic term, while the gauge vector has acquired mass. Moreover, $\hat{A}$ is decoupled (except for the factor $e^{-2\varphi}$) and the condition $\hat{A} = 0$ is consistent with the equations of motion. We can *excatly integrate* (by integrating we mean substituting the equations of motion) the massive mode, or, in other words, we have a *consistent truncation* of the theory. The remaining massless modes complete the sigma model with target space $\mathrm{SO}(1,n)/\mathrm{SO}(n)$.

From the geometric point of view, this is a special case. The contraction of $\mathfrak{solv}(\mathrm{SO}(1,1{+}n)/\mathrm{SO}(1{+}n))$ with respect to $\mathfrak{solv}(\mathrm{SO}(1,n)/\mathrm{SO}(n))$, does not change the algebra, so there is no a true contraction. No limit $\epsilon \to 0$, (related with taking the mass very big) has been necessary.

Inönü-Wigner contraction. We consider now the sigma model with target space $\mathrm{SU}(1,1+n)/\mathrm{U}(1+n)$. The solvable Lie algebra is

$$[H, Y_a] = Y_a, \quad [H, Z_a] = Z_a, \quad [Z_a, Y_b] = \delta_{ab}S,$$

with $a = 1, \dots n$.

We will take $n = 1$ for clarity. Using global solvable coordinates (s, z, y, ϕ), with coset representative

$$L(s, z, y, \phi) = \exp{(sS + zZ)}\exp{yY}\exp{\phi H},$$

we obtain the metric

$$\begin{aligned} ds^2 = 2d\phi^2 + \frac{1}{2}e^{-4\phi}dsds + e^{-4\phi}ydsdz + \\ \frac{1}{2}e^{-4\phi}(e^{2\phi} + y^2)dz^2 + \frac{1}{2}e^{-2\phi}dy^2, \end{aligned}$$

that makes manifest the translational isometries (Z, S). In this case we gauge the two translational isometries by introducing two gauge fields as before. The modes z, s are absorbed to give mass to the vectors, but now other interactions are present. Assuming that the mass of the vector fields is very big (the kinetic term is very small), we obtain algebraic equations for the gauge fields. After the elimination of the gauge fields using the approximate equations of motion, we obtain that the remaining modes ϕ, y) parametrize the manifold $\mathrm{SO}(1,2)/\mathrm{SO}(2)$

The same result can be obtained by performing a contraction of $\mathfrak{solv}(\mathrm{SU}(1,2)/\mathrm{SU}(2))$ with respect to $\mathfrak{solv}(\mathrm{SO}(1,2)/\mathrm{SO}(2))$. The result is $\mathfrak{solv}(\mathrm{SO}(1,2)/\mathrm{SO}(2)) \ltimes \mathbb{R}^2$, with metric

$$ds^2 = \left(2d\phi^2 + \frac{1}{2}e^{-2\phi}dy^2\right) + \frac{1}{2}e^{-4\phi}ds^2 + e^{-2\phi}dz^2.$$

The terms inside the parenthesis reproduce the metric of $\mathrm{SO}(1,2)/\mathrm{SO}(2)$. The other modes appear almost decoupled, in a way that it is consistent to truncate them to $z, s = 0$.

A we will see in next section, for more involved metrics it is not enough with a standard Inönü-Wigner contraction to obtain the simple structure of the above metric. Instead a generalized contraction may be needed.

Summarizing, we can say that the integration of the massive modes can be modeled by a group contraction followed by a quotient by an abelian invariant subgroup (or exact truncation).

4 A generalized contraction: application to Supergravity

We consider now the coset space $\mathrm{U}(2,1+n)/(\mathrm{U}(2)\times\mathrm{U}(1+n))$, with solvable Lie algebra

$$\begin{aligned}
&[Z^{ia}, Z^{jb}] = \epsilon^{ij}\delta^{ab}T^{(2,0)}\\
&[Y^{ia}, Y^{jb}] = \epsilon^{ij}\delta^{ab}T^{(0,2)}\\
&[Z^{ia}, Y^{jb}] = \delta^{ab}(\delta^{ij}S_2^{(1,1)} + \epsilon^{ij}S_1^{(1,1)})\\
&[Y^{ia}, S_1^{(1,-1)}] = Z^{ia}\\
&[Y^{ia}, S_2^{(1,-1)}] = \epsilon^{ij}Z^{ja}\\
&[T^{(0,2)}, S_\alpha^{(1,-1)}] = 2S_\alpha^{(1,1)}\\
&[S_\alpha^{(1,1)}, S_\beta^{(1,-1)}] = \delta_{\alpha\beta}T^{(2,0)}\\
&[H_+, Z^{ia}] = Z^{ia}\\
&[H_-, Y^{ia}] = Y^{ia},
\end{aligned}$$

with $i = 1,2, \quad \alpha = 1,2, \quad a = 1,\dots n$. The superindices indicate the weights with respect to the Cartan generators.

We consider the coordinates and coset representative

$$L(t,\tilde{t},\tilde{s}_\alpha, s_\alpha, z_{ia}, y_{ia}, \psi, \phi) = A(t,\tilde{t},\tilde{s}_\alpha, z_{1a})B(s_\alpha, z_{2a}, y_{ia})C(\psi,\phi) \qquad (1)$$

where

$$\begin{aligned}
A &= \exp\left(tT^{(2,0)} + \tilde{t}T^{(0,2)} + \tilde{s}_\alpha S_\alpha^{(1,1)} + z_{1a}Z^{1a}\right)\\
B &= \exp\left(s_1 S_1^{(1,-1)}\right)\exp\left(s_2 S_2^{(1,-1)}\right)\exp(z_{2a}Z^{2a})\exp(y_{2a}Y^{2a})\exp(y_{1a}Y^{1a})\\
C &= \exp\left(\psi H_+ + \phi H_-\right).
\end{aligned}$$

The metric is [1] (sum over repeated indices is understood, and we have used the short-hand notation $y_1^2 = y_{1a}y_{1a}$):

$$
\begin{aligned}
ds^2 &= d\phi^2 + d\psi^2 + e^{-4\psi}dtdt + 2e^{-4\psi}s_1 dtd\tilde{s}_1 + 2e^{-4\psi}s_2 dtd\tilde{s}_2 \\
&+2e^{-4\psi}z_{2a}dtdz_{1a} + 2e^{-4\psi}(s_2^2+s_1^2)dtd\tilde{t} + \frac{1}{2}(e^{-2(\psi+\phi)} + 2e^{-4\psi}s_1^2)d\tilde{s}_1 d\tilde{s}_1 \\
&+2e^{-4\psi}s_2 s_1 d\tilde{s}_1 d\tilde{s}_2 + \frac{1}{2}e^{-2(\psi+\phi)}(y_1^2+y_2^2)d\tilde{s}_1 ds_2 + e^{-2(\psi+\phi)}y_{1a}y_{2a}d\tilde{s}_1 ds_1 \\
&-e^{-2(\psi+\phi)}y_{1a}d\tilde{s}_1 dz_{2a} + (2e^{-4\psi}s_1 z_{2a} + e^{-2(\psi+\phi)}y_{2a})d\tilde{s}_1 dz_{1a} \\
&+2e^{-4\psi}s_1(e^{2(\psi-\phi)} + s_2^2 + s_1^2)d\tilde{s}_1 d\tilde{t} + \frac{1}{2}(e^{-2(\psi+\phi)} + 2e^{-4\psi}s_2^2)d\tilde{s}_2 d\tilde{s}_2 \\
&+e^{-2(\psi+\phi)}y_{1a}y_{2a}d\tilde{s}_2 ds_2 - \frac{1}{2}e^{-2(\psi+\phi)}(y_1^2+y_2^2)d\tilde{s}_2 ds_1 \\
&+e^{-2(\psi+\phi)}y_{2a}d\tilde{s}_2 dz_{2a} + (2e^{-4\psi}s_2 z_{2a} + e^{-2(\psi+\phi)}y_{1a})d\tilde{s}_2 dz_{1a} \\
&+2e^{-4\psi}s_2(e^{2(\psi-\phi)} + s_2^2 + s_1^2)d\tilde{s}_2 d\tilde{t} \\
&+\frac{1}{8}e^{-2(\psi+\phi)}(4e^{4\phi} + 4e^{2\phi}(y_1^2+y_2^2) + 4(y_{1a}y_{2a})^2 + (y_1^2+y_2^2)(y_1^2+y_2^2))ds_\alpha ds_\alpha \\
&-\frac{1}{2}e^{-2(\psi+\phi)}(2e^{2\phi}y_{1b} + (-2(y_{1a}y_{2a})y_{2b} + (y_1^2+y_2^2)y_{1b}))ds_2 dz_{2b} \\
&+\frac{1}{2}e^{-2(\psi+\phi)}(2e^{2\phi}y_{2b} + (2(y_{1a}y_{2a})y_{1b} + (y_1^2+y_2^2)y_{2b}))ds_2 dz_{1b} \\
&+e^{-2(\psi+\phi)}(y_2^2 s_1 + 2y_{2a}s_2 y_{1a} + s_1 y_1^2)ds_2 d\tilde{t} \\
&-\frac{1}{2}e^{-2(\psi+\phi)}(2e^{2\phi}y_{2b} + (2(y_{1a}y_{2a})y_{1b} + (y_1^2+y_2^2)y_{2b}))ds_1 dz_{2b} \\
&-\frac{1}{2}e^{-2(\psi+\phi)}(2e^{2\phi}y_{1b} + (-2(y_{1a}y_{2a})y_{2b} + (y_1^2+y_2^2)y_{1b}))ds_1 dz_{1b} \\
&-e^{-2(\psi+\phi)}(y_1^2 s_2 - 2y_{1a}s_1 y_{2a} + s_2 y_2^2)ds_1 d\tilde{t} \\
&-\frac{1}{2}e^{-2(\phi+\psi)}\epsilon_{ij}\epsilon_{mn}(y_{ia}y_{jb})dz_{ma}dz_{nb} \\
&+\frac{1}{2}e^{-2(\psi+\phi)}(e^{2\phi}\delta_{ab} + (y_{1a}y_{1b} + y_{2a}y_{2b}))dz_{2a}dz_{2b} \\
&-e^{-2(\psi+\phi)}(2y_1 s_1 - 2s_2 y_2)dz_2 d\tilde{t} \\
&+\frac{1}{2}e^{-4\psi}(e^{2\psi}\delta_{ab} + 2z_{2a}z_{2b} + e^{2(\psi-\phi)}(y_{1a}y_{1b} + y_{2a}y_{2b}))dz_{1a}dz_{1b} \\
&+(2e^{-4\psi}(s_2^2+s_1^2)z_{2a} + 2e^{-2(\psi+\phi)}(y_{1a}s_2 + s_1 y_{2a}))dz_{1a}d\tilde{t} \\
&+e^{-4(\psi+\phi)}(e^{2\psi} + e^{2\phi}(s_1^2+s_2^2))^2 d\tilde{t}d\tilde{t} - 2e^{-4\phi}y_{1a}d\tilde{t}dy_{2a} \\
&+\frac{1}{2}e^{-4\phi}(e^{2\phi}\delta_{ab} + 2y_{1a}y_{1b})dy_{2a}dy_{2b} + \frac{1}{2}e^{-2\phi}dy_{1a}dy_{1a}
\end{aligned}
$$

The generators $\{T^{(2,0)}, T^{(0,2)}, S_\alpha^{(1,1)}, Z^{1a}\}$ are translational isometries.

[1]This has been computed using the program Wolfram Research, Inc., Mathematica, Version 5.1, Champaign, IL (2004).

We have the following chain of symmetric spaces and their corresponding solvable Lie algebras:

$$\frac{\mathrm{SO}(1,1+n)}{\mathrm{SO}(1+n)} \subset \frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1+n)} \subset \frac{\mathrm{SO}(2,2+n)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n)} \subset \frac{\mathrm{SU}(2,2+n)}{\mathrm{U}(2)\times\mathrm{SU}(2+n)}$$

In fact $\mathfrak{solv}(\mathrm{SU}(1,1+n)/\mathrm{U}(1+n))$ can be embedded in more than one way as a subalgebra of $\mathfrak{s}_1 = \mathfrak{solv}(\mathrm{SU}(2,2+n)/(\mathrm{U}(2)\times\mathrm{SU}(2+n))$. For example,

$$\begin{aligned}\mathfrak{s}_2 &= \mathrm{span}\{H_+, Z^{ia}, T^{(2,0)}\},\\ \mathfrak{s}_2' &= \mathrm{span}\{H_-, Y^{ia}, T^{(0,2)}\}\end{aligned}$$

are both isomorphic to $\mathfrak{solv}(\mathrm{SU}(1,1+n)/\mathrm{U}(1+n))$, but they do not commute. Consequently, $\big(\mathrm{SU}(1,1+n)/\mathrm{U}(1+n)\big)^2$ is not a subgroup of $\mathrm{SU}(2,2+n)/(\mathrm{U}(2)\times\mathrm{SU}(2+n))$.

But there is a generalized contraction of $\mathfrak{s}_1$ in which $\mathfrak{s}_2\oplus\mathfrak{s}_2'$ is a subalgebra. To perform the contraction we split $\mathfrak{s}_1 = \mathfrak{g}_0+\mathfrak{g}_1+\mathfrak{g}_2$, with

$$\begin{aligned}\mathfrak{g}_0 &= \mathrm{span}\{H_+, H_-\},\\ \mathfrak{g}_1 &= \mathrm{span}\{Y^{ia}, Z^{ia}, S_\alpha^{(1,1)}\},\\ \mathfrak{g}_2 &= \{T^{(0,2)}, T^{(2,0)}, S_\alpha^{(1,-1)}\},\end{aligned}$$

and the linear map is

$$\mathfrak{s}_4 \longrightarrow \mathfrak{s}_4 e_0 + e_1 + e_2 \longrightarrow e_0 + \epsilon e_1 + \epsilon^2 e_2, \qquad e_i \in \mathfrak{g}_i.$$

The deformed algebra is

$$\begin{aligned}
&[Z^{ia}, Z^{jb}]_\epsilon = \epsilon^{ij}\delta^{ab}T^{(2,0)}\\
&[Y^{ia}, Y^{jb}]_\epsilon = \epsilon^{ij}\delta^{ab}T^{(0,2)}\\
&[Z^{ia}, Y^{jb}]_\epsilon = \epsilon\ \delta^{ab}(\delta^{ij}S_2^{(1,1)} + \epsilon^{ij}S_1^{(1,1)}) \rightarrow 0\\
&[Y^{ia}, S_1^{(1,-1)}]_\epsilon = \epsilon^2\ Z^{ia} \rightarrow 0\\
&[Y^{ia}, S_2^{(1,-1)}]_\epsilon = \epsilon^2\ \epsilon^{ij}Z^{ja} \rightarrow 0\\
&[T^{(0,2)}, S_\alpha^{(1,-1)}]_\epsilon = \epsilon^3\ 2S_\alpha^{(1,1)} \rightarrow 0\\
&[S_\alpha^{(1,1)}, S_\beta^{(1,-1)}]_\epsilon = \epsilon\ \delta_{\alpha\beta}T^{(2,0)} \rightarrow 0\\
&[H_+, Z^{ia}]_\epsilon = Z^{ia}\\
&[H_-, Y^{ia}]_\epsilon = Y^{ia}
\end{aligned}$$

In the contraction limit, the metric becomes

$$
\begin{aligned}
& ds^2 = \big(d\phi^2 + e^{-4\phi} d\tilde{t} d\tilde{t} - 2e^{-4\phi} y_{1a} d\tilde{t} dy_{2a} + \\
& \frac{1}{2} e^{-4\phi} (e^{2\phi}\delta_{ab} + 2y_{1a}y_{1b}) dy_{2a} dy_{2b} + \\
& \frac{1}{2} e^{-2\phi} dy_{1a} dy_{1a}\big) + \\
& \\
& \big(d\psi^2 + e^{-4\psi} dt dt + 2e^{-4\psi} z_{2a} dt dz_{1a} + \\
& \frac{1}{2} e^{-4\psi} (e^{2\psi}\delta_{ab} + 2z_{2a}z_{2b}) dz_{1a} dz_{1b} + \\
& \frac{1}{2} e^{-2\psi} dz_{2a} dz_{2a}\big) + \\
& \\
& + \frac{1}{2} e^{-2(\psi+\phi)} d\tilde{s}_\alpha d\tilde{s}_\alpha + \frac{1}{2} e^{-2(\psi+\phi)} ds_\alpha ds_\alpha .
\end{aligned}
$$

The modes $s_\alpha, \tilde{s}_\alpha$ can be exactly set to zero bu using its field equations, so a trivial truncation of this theory gives a sigma model on

$$
\frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1+n)} \times \frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1+n)}
$$

4.1 Supergravity interpretation

We consider an $N=2$ supergravity model coupled to $n+2$ hypermultiplets (maximal helicity 1/2) and $n+1$ vector multiplets (maximal helicity 1).

We are interested in a particular model which has a ten dimensional origin. If we consider type IIB SUGRA compactified on the orientifold $T^6/\mathbb{Z}^2$ we obtain an $N=4$ theory. When certain fluxes of the forms in ten dimensions are turned on, this theory has an $N=3$ phase, obtained after the integration of a massive gravitino multiplet. Turning on other fluxes and performing further integration we arrive to an $N=2$ phase, which is the object of our interest.

The scalar manifold for this theory is

$$
\mathcal{M}_Q \times \mathcal{M}_{SK} = \frac{\mathrm{U}(2,2+n)}{\mathrm{U}(2)\times \mathrm{U}(2+n)} \times \frac{\mathrm{U}(1,1+n)}{\mathrm{U}(1)\times \mathrm{U}(1+n)}.
$$

(Here n refers to the brane degrees of freedom.) It is the product of a quaternionic manifold $\mathcal{M}_Q$(hypermultiplets) times a special Kälher manifold $\mathcal{M}_{SK}$ (vector multiplets).

We can gauge the translational isometries of the quaternionic manifold generated by $S_\alpha^{(1,1)}$ using two bulk vector fields. The modes $\tilde{s}_\alpha$ disappear to give mass to the vectors. The fields s_α also acquire a mass through the potential induced by the gauging. In the large mass limit, these fields can be set to zero

It can be shown that after the gauging and the integration of the massive modes, the metric becomes the one of

$$\frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1+n)} \times \frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1+n)} \times \frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1+n)},$$

where $\mathcal{M}_{SK}$ has not been touched [5]

We see now that the terms set to zero in the metric by taking the limit $\epsilon \to 0$ are precisely the terms eliminated by the integration procedure, irrespectively if it is through a Higgs mechanism or because they acquire mass through the potential.

The contraction procedure seems a more general mechanism than the integration. It remains open the interpretation of other contractions.

In the case of supersymmetric theories we are interested in curves in the space of metrics that have two or more supersymmetric points. These form a smaller set and it is perhaps easier to find a full interpretation of the mechanism.

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